LA-UR 88-1734

CONF-880320--1

LA-UR--88-1734

DE88 010931

TITLE COUPLED-BUNCH INSTABILITIES IN RHIC

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SUBMITTED TO Proceedings of the Workshop on RHIC Performance
Brookhaven National Laboratory
Upton, Long Island, New York
March 21-26, 1988

Los Alamos National Laboratory
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1 Introduction

During the RHIC workshop we were only able to consider the case of gold ions. We used the program ZAP[1] to evaluate the coupled bunch modes for the case of 57 equally spaced bunches with the Sacherer Zotter formalism assuming parabolic bunches. The sources of coupling impedances assumed were space charge, resistive wall, broadband, and rf cavity fundamental and parasitic modes. Generally the studies assumed a stainless steel vacuum chamber, but we did perform a comparison run using a stainless steel chamber internally coated with a thin cold copper layer. These latter investigations were motivated by the proposal for coating which would reduce parasitic wall heating in the vacuum chambers in the superconducting dipoles.

Investigations were carried out at both injection (kinetic energy of 10.7 GeV/amu)

Table 1: Fixed Machine Parameters (RHIC)

Circumference	3833.87 m
Betatron Tunes Q_x, Q_y	28.826, 28.822
Average β_x, β_y	55.5 m
Beam Pipe Radius	36 mm
Transition Gamma	25.4
Ion Charge	79
Ion Atomic Mass Number	197
Mass per a m	0.93126 GeV
Number of Bunches	57
Number Ions per Bunch	10^9

and at final energy (kinetic energy of 100 GeV/amu) for gold. For the former case we used the design harmonic 342 rf system consisting of six cavities, each providing 200 kV. Higher order modes were provided during the workshop and they were incorporated in the runs. At the final energy we considered both the $h = 342$ system, and a proposed $h = 2736$ system with higher-order modes taken from measurements at CERN;[2] in this latter case the net rf voltage was 15 MV. In Table 1 we list the energy independent parameters used by ZAP.

2 The Effect of a Cold Copper Coating on The Inside of The Stainless Steel Chamber

Studies of transverse resistive wall instability were carried out solely at the injection kinetic energy (10.7 GeV/amu). The machine was unstable for the rigid dipole modes ($a = 0$) at the lowest frequencies $f = (n k_b + s + Q_y) f_0$ where k_b = the number of bunches (57), $n = 1$, $s = -28$ and f_0 is the revolution frequency of 78 kHz. Table 2 lists the fastest growth rates for (a) a stainless steel chamber, and (b) a stainless steel chamber with a thin (25 micron) deposit of cold copper on the inner wall. As usual the growth rate ($1/\tau$) is the imaginary part of the complex frequency shift $\Delta\omega^{\perp}_{ca}$. The explicit formulas for the frequency shifts are presented in the appendix.

Table 2: Transverse Resistive-Wall Growth Rates

s	$\frac{f}{f_0}$	$1/t$ (sec^{-1}) ss	$1/t$ (sec^{-1}) ss with cc
28	0.178	23.66	0.66
27	1.178	9.04	0.25
26	2.178	6.57	0.18
25	3.178	5.35	0.15
24	4.178	4.58	0.13

The program ZAP assumes a wall thickness of one skin depth. One should use the minimum of the skin depth or the actual wall thickness (1.65 mm); therefore the growth rates for $s = 28$ (appearing in Table 2) have to be increased by about a factor of 1.8. Thus the modified growth rates for $s = 28$ are 43 sec^{-1} for stainless steel and 1.32 sec^{-1} for a cold copper coated chamber. We realize a factor of approximately 33 improvement by using a cold copper coating on the inside of the vacuum pipe around the entire circumference.

3 Studies of the Transverse Coupled-Bunch Instabilities

In all cases we took the vacuum chamber to be stainless steel throughout in RHIC and to have a beam pipe radius of 36 mm. Contributions to the coupling impedance were taken to arise from space charge, resistive wall, broadband, and higher-order parasitic cavity modes. The higher order modes as calculated by URMEL[3] are listed for the six h = 342 cavities in Table 3; these are the dominant parasitic modes.

The chromaticities were set to zero in the ZAP runs. Initially we considered the case at injection energy. The parameters for the runs are the rms geometric emittance area $1.33\,\pi \times 10^{-7}$ m, the rms relative momentum spread σ_p of 6.8×10^{-4} and the rms bunch length σ_ℓ of 0.61 m. The small amplitude synchrotron tune Q_s is defined by $|\eta| R \sigma_p / \sigma_\ell$ where $\eta = \gamma_t^{-2} - \gamma^{-2}$ and R is the average radius of the machine. We find $Q_s = 3.3 \times 10^{-4}$.

Table 3: Transverse Parasitic Modes Used in ZAP for the h=342 System

$\omega_r(10^6 radians/sec)$	RT (Mohm/m)	Q
2851	2.04	15900
3123	3.60	15600
3401	5.40	14700
3666	6.30	13800
3899	6.00	12600
4132	7.50	13800
4408	9.60	15600
4707	10.50	16200
4974	7.20	14700
5366	1.62	17100

It turned out that the motion was unstable with growth rates as given above in Table 2 for stainless steel. The resistive wall contributions are dominant. The space charge term gives rise to a significant real frequency shift Re $\Delta\omega^1_{s,0} = 1.63 \times 10^4$. The Landau damping calculation is not performed in ZAP for parabolic bunches. Therefore we redid the calculations assuming a Gaussian bunch using the Wang formalism; if we include a nonlinear tune spread σ_Q of 5×10^{-2}, then the rigid dipole motion is stabilized for all frequencies. This condition is simply $| \Delta\omega^1_{s,a} | < 0.7776\, \omega_0\sigma_Q$. We cannot rely on tune spread from a nonzero chromaticity to stabilize the beam because the time for one synchrotron oscillation is 3.9 msec, much shorter than the fastest growth time of 42 msec (see Table 2). The tune spread quoted above (5×10^{-2}) is a bit excessive. The preferred solution is a low-frequency transverse damper.

Identical studies were carried out at the final kinetic energy of 100 GeV/amu using the h = 342 rf system with 1.2 MV/turn and the higher-order parasitic modes given in Table 3. In this case the rms geometric emittance area was $1.54\,\pi \times 10^{-8}$ m, σ_p was 3.3×10^{-4}, σ_ℓ was 0.178 m, and Q_s was 0.62×10^{-3}. The growth rates and real frequency shifts for the fastest instabilities are presented in Table 4. Again these are for the rigid dipole mode only. The values for the growth rates at s = 28 should be multiplied by 1.8 (see above).

The situation at higher energy is improved over the results seen at the injection

Table 4: Transverse Coupled-Bunch Instability Growth Rates at 100 GeV/amu with h = 342 System

s	$\frac{f}{f_0}$	Re $\Delta\omega^{\perp}_{s,0}$ (1/sec)	1/t (1/sec)
28	0.178	185	2.73
27	1.178	186	1.04
26	2.178	187	0.76
25	3.178	187	0.62
24	4.178	187	0.53

kinetic energy of 10.7 GeV/amu the growth times for instability are a factor of 8 2/3 longer and the real frequency shifts are down significantly. In fact, stabilization can be achieved with an rms tune spread σ_Q of 0.5×10^{-3}. Of course, we might as well utilize the transverse damper which appears to be required at the injection energy.

As stated above we explored the situation with the h = 2736 rf system operating with 15 MV. The higher-order parasitic modes were assumed to be identical to those of the CERN design which incorporated 200 MHz cavities with two coaxial higher-order mode couplers.[7] Table 5 lists the transverse parasitic modes used in the ZAP calculations (assuming 15 cavities with 1 MV each).

The values of the shunt resistances represent the most pessimistic values expected, namely that all cavities are identical. The ZAP runs were carried out for the h = 2736 system. The h = 342 system is assumed to be transparent (shorted out?) in these studies. As before we consider impedance contributions due to space charge, resistive wall and the parasitic cavity modes given in Table 5. We also included a broadband Q = 1 resonator with peak $Z_{\|}/n$ = 10 ohms; the peak is at an angular frequency $\omega = c/b$ where b is the beam pipe radius (36 mm). The conversion to a transverse impedance is performed via the usual relation $Z_{\perp} = (Z_{\|}/n) \cdot 2R/b^2$. The new longitudinal parameters used were σ_p = 1.055 × [illegible], σ_ℓ = 0.149 m, and Q_s = 6.33 × [illegible]. The results for the growth rates were essentially identical to those shown in Table 4. However the real frequency shifts obtained were of the order of 600; the run was repeated for Gaussian bunches with resulting frequency shifts of the order of 175. An rms nonlinear tune spread σ_Q of approximately 1.25 × [illegible] is sufficient to provide Landau damping.

Table 5: Transverse Parasitic Modes Used in ZAP for the h = 2736 System

ω_r ($\times 10^6$ radians/sec)	RT (Mohm/m)	Q
2482	25.65	1900
3211	17.55	2600
3280	8.55	1900
3625	1.33	890
4254	8.10	3600
4279	13.65	1400
4442	3.82	5100
4769	0.82	1100

4 Studies of the Longitudinal Coupled-Bunch Instabilities

The analysis closely parallels that given above for the transverse modes. The bunches are assumed to be parabolic with the complex frequency shifts being calculated using the Sacherer-Zotter formalism. The imaginary part of the frequency shift is the growth rate of the instability ($1/t$). The relevant formulae are contained in the appendix. Contributions to the longitudinal coupling impedance were assumed to arise from a stainless steel resistive wall throughout, longitudinal space charge, the rf cavity fundamental, an equivalent broadband $Q = 1$ resonator with angular frequency $\omega_r = c/b$, and assumed rf cavity parasitic modes. The broadband impedance $Z_{\parallel}/n$ was chosen for each run. The possible frequencies of unstable oscillations are $f = (p\,k_b + s + a\,Q_s)f_0$ where k_b is the number of bunches (57), p is any integer, s is an integer $0 \leq s \leq k_b - 1$, Q_s is the small amplitude synchrotron tune and a is the oscillation mode; a = 1 for dipole motion, a = 2 for quadrupole motion, etc.

The first runs were at the injection kinetic energy of 10.7 GeV/amu for gold. The fundamental rf system was assumed (h = 342) with an initial rf frequency of 26.657 MHz. The input parameters are as in Table 1 with those used above for the transverse case: rms geometric emittance area of $1.33 \times 10^{[illegible]}\pi$ m, $\sigma_p = 6.8 \times 10^{-4}$, and $\sigma_\ell = 0.61$ m. The

Table 6: Longitudinal Parasitic Modes Used in ZAP for the h=342 System

ω_r ($\times 10^6$ radians/sec)	R_s (Mohms)	Q
508	5.85	5633
856	2.79	6933
1192	1.46	7730
1830	0.95	8767
1847	0.79	10433
2204	0.69	12120
2579	0.60	13400
2959	0.55	14333
3337	0.51	14867
3698	0.47	14600

total shunt resistance for the rf fundamental was 16.62 Mohm with a loaded Q = 3967. The higher-order parasitics were obtained from the URMEL run described above — they are listed in Table 6 (assuming six cavities carrying 200 kV each at the fundamental).

The results of the ZAP runs are presented in Table 7 for the modes with the largest growth rates for longitudinal coupled bunch instabilities. The results are shown only for coherent dipole motion (a = 1); the broadband $Z_{\parallel}/n$ was taken to be 0 ohms for this case.

Table 7: Longitudinal Coupled Bunch Instability Results at Injection Energy in RHIC

s	Re $\Delta\omega_{s,1}$ (1/sec)	[illegible] (1/sec)
16	77.3	17.95
16	[illegible]	31.62
18	107.3	22.01
15	100.1	[illegible]
19	[illegible]	[illegible]

Table 8: Longitudinal Coupled-Bunch Instability Results at 100 GeV/amu Using the h = 342 System

s	Re $\Delta\omega^{\parallel}_{A,1}$ (1/sec)	1/t (1/sec)
8	-4.60	70.4
33	27.11	23.8
55	-5.21	3.1
34	-11.18	1.6
54	10.22	1.5

The instability results for the five fastest frequencies indicate that the beam is generally unstable with worst case growth times of approximately 21 msec. Furthermore we have rerun ZAP with just one parasitic resonator but with variable frequency so as to lock on directly to an integral multiple of the revolution frequency; we used the first one listed in Table 6. The fastest mode had s = 46 and the growth rate increased from 31.62 to 340.1. Thus we can expect that in the worst cases some of the growth times could be up to 10× worse than the values shown in Table 7. Unfortunately the bunch length is too small to provide enough synchrotron tune spread for Landau damping of the instabilities. The test for Landau damping used in ZAP requires that

$$| \Delta\omega^{\parallel}_{s,1} | \quad \frac{1}{16}\left(\frac{\sigma_\ell}{R}\right)^2 h^2 Q_s \omega_o$$

for the coherent dipole mode for the present conditions this means $| \Delta\omega^{\parallel}_{s,1} |$ · 5.9 for damping to occur. For satisfactory operation feedback would be required as well as running in between the parasitic frequencies.

Moving up to the top kinetic energy of 100 GeV/amu, we repeated the same studies with the same rf system including the parasitic modes listed in Table 6. This time the rms geometric emittance was taken to be $1.51 \cdot 10^{-8}\ \pi$ m, σ_p $3.3 \cdot 10^{-4}$, σ_ℓ 0.478 m, and the resulting synchrotron tune Q_s 0.62 10^{-3} for a peak voltage of 1.2 MV. The results for the rigid dipole motion are presented in Table 8. As above we present the cases for the five fastest growth rates. The broadband $Z_{\parallel}/n$ was taken to be 0 ohms.

Table 9: Longitudinal Parasitic Modes Used in ZAP for the h-2736 System

ω_r ($\times 10^6$ radians/sec)	R_s (Mohm)	Q
1923	0.960	4000
2802	0.165	1400
3393	0.060	540
3764	1.515	7200
4725	0.008	520
5020	0.210	1100
5328	0.075	4700
6183	2.355	26200

We see that the motion is again unstable, due primarily to the parasitic modes with fastest growth time of the order of 14 msec. The first resonator listed in Table 6 is directly responsible for this case: The frequency of the resonance being f=(18*57 + 8)* f_0. As we saw above for the injection energy, there is still insufficient synchrotron frequency spread to provide Landau damping against these instabilities; the ratio of the areas of the bunch to bucket are just 0.1 at the end of acceleration. The requirement for Landau damping is $|\Delta\omega_{s,1}| \leq 1.37$, but the computed value of $|\Delta\omega_{s,1}|$ is 74 for the worst case (s=8).

Next we consider the longitudinal stability for inclusion of the high frequency rf system, which is the h=2736 system at the top energy. The fundamental rf frequency is 213.933 MHz and the assumed net voltage is 15 MV (peak); the total shunt resistance is 130.575 Mohm and the Q is 46800. We tacitly assume that the low frequency rf system is transparent (nonexistent) for these calculations. The higher-order parasitic modes are adapted from the CERN work[7] and are listed in Table 9 (the numbers refer to 15 cavities with 1 MV each).

The values of the shunt resistances again represent the pessimistic case where all cavities are identical. The ZAP runs were carried out as described above and we first performed the runs assuming a broadband $Z_{\|}/n$ of 0 ohms. The results for the rigid dipole motion (a=1) for the five fastest growth rates are listed in Table 10.

Table 10: Longitudinal Coupled Bunch Instability Results at 100 GeV/amu Using the h=2736 rf System

s	Re $\Delta\omega^{\parallel}_{s,1}$ (1/sec)	1/t (1/sec)
23	72.8	4.3 (d)
38	-0.8	3.4 (d)
24	-5.0	2.4 (d)
45	-1.6	1.6 (d)
22	0.2	0.9 (d)

All of the instabilities are damped (d) by synchrotron frequency spread by virtue of the increased ratio of bunch/bucket area. We increased the broadband $Z_{\parallel}/n$ in steps until the damping was lost because of increased values of Re($\Delta\omega_{s,1}$); when $Z_{\parallel}/n$ is in the range of 5-10 ohms (and above) the damping is lost. According to Table 10 the growth rates for the rigid dipole modes are of the order of 17 times smaller than those for the case of the h=342 rf system.

5 Discussion

The comments which follow are strictly true for RHIC with a fill of 5.7×10^{10} gold ions; a repeat of our analysis with protons needs to be performed.

With regard to transverse coupled-bunch instabilities it appears that the worst case growth rates are generated by the resistive wall instability. A low frequency transverse damper is recommended for stabilization. Use of a cold-copper coating on the inside of the stainless steel vacuum chamber reduces the growth rates by a factor of roughly 30 if we assume a coated chamber around the entire circumference.

The situation in regard to the longitudinal coupled-bunch instabilities is not quite so clear; assumption of parasitic modes for as yet unbuilt cavities entails some amount of uncertainity. Nevertheless one has to respect the potential harm of these narrow band "resonators." One should endeavor to keep the broadband $Z_{\parallel}/n$ to a low value for the ring, say of the order of a few ohms. There appears to be a negative aspect of using the h=342 system of six cavities with 200 kV each as described in the RHIC proposal, namely that the ratio of bunch to bucket area is too small to provide Landau damping against the worst instabilities. More recent developments may change this scenario in the future and these coupled bunch analyses will have to be repeated. The higher frequency rf system affords ample synchrotron frequency spread so as to Landau damp the worst modes, as long as we maintain a modest strict impedance budget. In any case we should assume that some longitudinal dampers will be necessary in RHIC.

6 Acknowledgment

We thank Dr. P. Schoessow of Argonne especially for providing the higher-order parasitic modes for the RHIC cavities. The direction of the working group chairman, Mike Zisman, was instrumental in achieving the results presented in this paper. Lastly we thank the staff of the Brookhaven National Laboratory for making our stay a very pleasant experience and for providing whatever help was required.

7 References

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APPENDIX

Calculation of the Complex Frequency Shifts

All of the information in this appendix was obtained from the ZAP users manual[1].

- Longitudinal Coupled-Bunch Instability:

  The various impedances in the ring, especially the narrow-band (high Q) resonances of the rf cavity parasitic modes will excite coupled-bunch normal modes of the beam. The unperturbed modes of the beam have angular frequency $\omega_p = (p\,k_b + s + a\,Q_s)\omega_0$ where p is any integer, k_b is the number of bunches (57 for RHIC), s is the longitudinal mode number $0 \leq s \leq k_b-1$, a represents the motion in longitudinal phase space (a = 1 for the rigid-dipole mode, a = 2 for quadrupole, etc), and Q_s is the small amplitude synchrotron tune. The revolution angular frequency $\omega_0 = \beta c/R$ where R is the effective machine radius (610.18 m). In the beam frame the frequency of mode (s,a) is $aQ_s\omega_0 = a\omega_s$. Interaction of the beam with the ring longitudinal impedance $Z_{\parallel}(\omega)$ causes a complex frequency shift $\Delta\omega_{s,a}$. The real part of $\Delta\omega_{s,a}$ is the real coherent frequency shift. If the imaginary part of $\Delta\omega^{\parallel}_{s,a}$ is positive, then the beam is unstable with growth rate equal to $\mathrm{Im}(\Delta\omega^{\parallel}_{s,a})$.

Following the ZAP users manual we write the complex frequency shift

$$\Delta\omega^{\parallel}_{s,a} = i\left(\frac{a}{a+1}\right)\frac{q_i I_b \omega_0^2 \eta}{3(L/2\pi R)^3 2\pi\beta^2 (E_t/e)\omega_s}\cdot\left[\frac{Z_{\parallel}}{n}\right]^{s,a}_{eff} \tag{1}$$

where I_b is the bunch average electrical current ($I_b = N_b q_e f_0$), $\eta = \gamma_t^{-2} - \gamma^{-2}$, L is the total length of a bunch ($L = 2\sqrt{2}\,\sigma_\ell$ a parabolic bunch), E_t is the total relativistic energy of the ion in question, and the effective longitudinal impedance calculated as:

$$\left[\frac{Z_{\parallel}}{n}\right]_{eff}^{s,a} = \frac{\sum_{p=-\infty}^{+\infty} \frac{Z_{\parallel}(\omega_p^{\parallel})}{(\omega_p^{\parallel}/\omega_0)} h_a(\omega_p^{\parallel})}{\sum_{p=-\infty}^{+\infty} h_a(\omega_p^{\parallel})} . \tag{2}$$

We use the Sacherer-Zotter formalism with this approach. The spectral power densities are given by

$$h_0(\omega) = (a+1)^2 \frac{\left[1 + (-1)^a \cos\left(\omega \frac{L}{\beta c}\right)\right]}{\left[\left(\frac{\omega L}{\pi \beta c}\right)^2 - (a+1)^2\right]^2} . \tag{3}$$

The impedances indicated in Eq. A2 are the standard ones appropriate to space charge, resistive wall, broadband, rf cavity fundamental and high Q resonators.

- Transverse Coupled Bunch Instability:

  In this case we restrict ourselves to the rigid-dipole motion only ($a = 0$). The unperturbed modes, when observed in the laboratory at a fixed location, will have angular frequencies given by $\omega_p^{\perp} = (p\, k_b + s + Q_y)\omega_0$ where Q_y represents the transverse betatron tune. In the beam frame the frequency of mode s,a will be $\omega_\beta = Q_y \omega_0$. The perturbed frequency of coherent oscillation is modified from the unperturbed value by the complex frequency shift $\Delta\omega_{s,0}^{\perp}$ caused by the ring transverse impedance $Z_{\perp}(\omega_p^{\perp})$. The frequency shift is given (for a parabolic bunch in the Sacherer Zotter formalism) by

$$\Delta\omega_{s,0}^{\perp} = i \frac{q_i I_b \beta c^2}{2\omega_\beta (E_T/e) L} \cdot [Z_{\perp}]_{eff}^{s,0} \tag{4}$$

where the effective transverse impedance is obtained using

$$[Z_{\perp}]_{eff}^{s,0} = \frac{\sum_{p=-\infty}^{+\infty} Z_{\perp}(\omega_p^{\perp}) h_0(\omega_p^{\perp} - \omega_\xi)}{\sum_{p=-\infty}^{+\infty} h_0(\omega_p^{\perp} - \omega_\xi)} . \tag{5}$$

The quantities in Eq. (5) are given by

$$\omega_\xi = \frac{\xi}{\eta} \omega_0$$

and

$$\xi \quad Q_y/(dp/p)$$

Here $h_0(\omega)$ is the same bunch mode spectrum defined for the longitudinal case, except that it is shifted by the chromatic frequency ω_ξ.